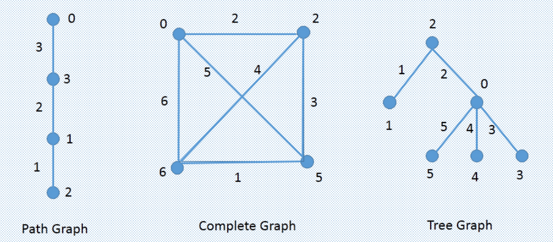
Graceful Labeling of a Special Class of tree

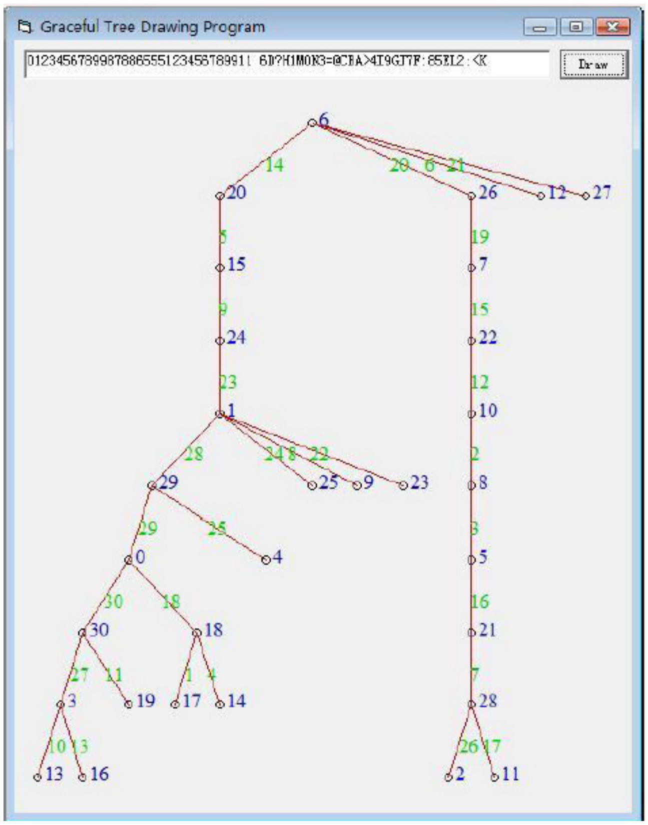
Sakib Shafayat, Khan Sabah Syedee, Md. Rezwanur Rahman

Graceful Labeling of Trees

Let G = (V,E) be a graph with n vertices and m edges, and let f: V → N be a labeling of the vertices of V . Let a_i = $f(v_i)$, for 1≤i ≤ n. Define $V_{\varphi G}$={a_1, a_2, ..., a_n,} to be the set of labels of the vertices of G given by f. For any edge e_k= $v_i v_j$ in E, let b_k= |$f(v_i)$-$f(v_j)$| = |a_i-a_j|. We denote by $E_{\varphi G}$ the set of values b_k, of the edges of G induced by f.



- **AI approach:** By applying constraints and methods of AI we can gracefully label any tree. The recent effort in this method has proved that a tree with 35vertices are graceful. But the computational resource needed in this approach is still too much to prove the conjecture true.

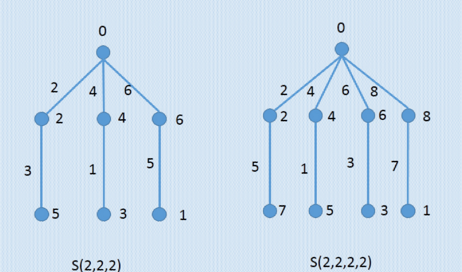


- This approach uses Deterministic Backtracking and Probabilistic Search to solve trees with 35 vertices. We know that the number of trees with n vertices grows exponentially with n. An empirical relation is O(2.6687^n). So computational power needed for verification also grows exponentially.

Special class of Graceful Trees

- ***Lemma:*** Let f_1 be a bipartite labeling with labels starting at 0 of a tree S with f_1(u) = 0 for some u ∈ V(S). Let f_2 be a graceful labeling (bipartite labeling, respectively) with labels starting at 0 of a tree T with f_2(v) = 0 for some v ∈ V (T). Then there is a graceful labeling (bipartite labeling, respectively) of S_u $join$ T_v.
- ***Theorem 1***: The spiders S(x_1, x_2, x_3) with three legs are graceful.
- **Proof:** This proof follows from the above Lemma. Identify a leaf from the path P_{x1} with a leaf of P_{x2} . The resulting tree, T, is itself a path. Choose a graceful labeling of T such that the identified vertex has label 0. Then, choose a bipartite labeling of P_{x3} such that one of its leaves has label 0. Finally, joining T and Px3 and relabeling according to the above Lemma results in the tree S(x_1, x_2 , x_3) ,which is labeled gracefully.

- ***Theorem 2***: The spiders S(x_1, x_2, x_3, x_4) with 4 legs are graceful.
- **Proof:** If at least one of x_1, x_2, x_3, x_4 is not 2, then without loss of generality $x_1 + x_2 \neq 4$. Let u be the central vertex of the spider S(x_1, x_2) = $P_{x_1+x_2}$ and v be the central vertex of the spider S(x_3, x_4) = $P_{x_3+x_4}$. We know that there is a bipartite labeling of S(x_1, x_2) that labels u with 0 and a graceful labeling of S(x_3, x_4) that labels v with 0. The result follows from Theorem 1. The only other possibility is that x_1= x_2 $= x_3$ $= x_4$ = 2. A graceful labeling of S(2; 2; 2; 2) is shown in Figure.



Graceful Labeling of Spider Graphs

Department of Computer Science and Engineering (CSE), BUET